A Tale of Two Slinkies: Learning about Model Building in a Student-Driven Classroom

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Calvin Berggren, Punit Gandhi, Jesse A. Livezey, and Ryan Olf

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Calvin Berggren, Punit Gandhi, Jesse A. Livezey, and Ryan Olf, Berkeley Compass Project, University of California, Berkeley, CA

We describe a set of conceptual and hands-on activities based around understanding the dynamics of a Slinky that is hung vertically and released from rest. This Slinky drop experiment typically lasts a fraction of a second, but when observed in slow motion, one sees the Slinky compress from the top down while the bottom portion remains at rest[1]—naively seeming to defy gravity—until the Slinky has completed its collapse. The motion, or lack thereof, of the bottom of the Slinky after the top is released sparks student curiosity by challenging expectations and provides motivation and context for learning about scientific model building.

The Slinky drop and other related phenomena have been studied in detail[2-8] and have attracted a flurry of interest online.[9] In this work, we describe how we used the Slinky drop to teach the modeling process by giving students first-hand experience constructing two distinct but complementary physical models: a discrete mass model and a continuous wave model. With two different models, students not only have the opportunity to understand an intriguing phenomenon from multiple perspectives, but also learn deeper lessons about the nature of scientific understanding, the role of physical models, and the experience of doing science. The sequence of activities we present were developed as part of a week-long summer program for incoming freshmen through the Berkeley Compass Project[10-14] but could easily be implemented in a wide range of classrooms at the high school and introductory college levels.

After providing a brief overview of the curriculum and the classroom setting in which it was implemented, we describe the key activities that allowed the students to explore each of the two models and provide concluding remarks.

Curriculum overview

The Slinky drop experiment strikes a delicate balance between competing pedagogical needs: the problem is both approachable and deep, allowing students to explore their conceptual frontier regardless of their physics background. Moreover, it lends itself well to models with disparate conceptual bases, providing an ideal backdrop for building and comparing scientific models. Table I outlines the four major parts of a curriculum based on the Slinky drop experiment that was implemented with 16 students over seven days. In addition to approximately 30 hours of instruction, the students spent about 15 hours outside of class on group homework assignments and final projects. The curriculum does not assume previous physics knowledge, and all the concepts from Newton's laws to wave propagation were introduced in a self-contained way in a collaborative learning environment that blends several proven inquiry- and discovery-based strategies.[15-18]

Table I. Summary of the four sections of the week-long program.

Section	Class Time	Important Concepts
Falling Slinkies vs. falling rigid objects	4 hours	Newton's second law, gravity, free-body diagrams
Mass and spring model	7 hours	Hooke's law, discrete approximation, numerical simulation
Wave/information model	7 hours	wave propagation, information
Final projects	10 hours	additional quantitative investigation of unresolved questions

In the first part of the course, the students reconciled their observations of the levitating Slinky with their existing intuition about gravity based on point objects. The key takeaway from this segment was that the Slinky need not defy gravity for part of it to remain stationary during free fall, so long as the center of mass behaves appropriately. The students experimentally determined the center of mass by finding the point along a vertically hung Slinky where a ball, if released simultaneously with the Slinky, hits the floor at the same time as the Slinky.[19]

Convinced that the laws of gravity apply, the students began to develop models that could explain the Slinky drop phenomenon in the next two parts of the course. The students first modeled the Slinky as a finite number of point masses connected by linear massless springs, allowing them to apply Newton's laws to calculate the motion of the masses. Observations made during the exploration of this model motivate a second approach wherein students modeled the propagation of information in the Slinky. This model describes the event when the Slinky is released as a piece of information that needs to travel to other parts of the system—via either a wave pulse or the shock wave of medium collapse—before it is able to respond.

The fourth and final phase of the curriculum involved group research projects where the students chose to explore a question related to the Slinky phenomena in depth and create a short video (accessible online[20]) explaining their results for a general audience.

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A discrete mass and spring model of the Slinky

In the "discrete model," students used forces and Newton's laws to calculate the motion of the Slinky in free fall. This model is conceptually straightforward and allowed students to follow a reductionist approach wherein they divide the Slinky into intuitively and mathematically accessible constituents. In particular, students modeled the Slinky as a series of point masses connected by massless springs that obey Hooke's law.[5] In order to glean insight into the Slinky drop, they investigated the roles that various parameters of the model—number, mass, spring constant—play in reproducing the relevant qualities of a Slinky. The students explored the model in two complementary ways: they built and studied a physical approximation of the discrete system, and they derived and numerically evolved equations of motion.

A. Experimental exploration

The students explored the applicability and limitations of the discrete model and the implications of the various choices involved in defining it by building physical realizations of the model using masses (washers, nuts) tied to springs (rubber bands, stretchy silicone). A sample physical model being tested in the classroom is shown in Fig. 1. The primary focus of student investigation was whether, and under what circumstances, the physical mass and spring models demonstrate the levitation effect that they sought to understand. To this end, students built many models with different masses and springs and noted the manner in which they dropped, as recorded by a high-speed camera. From these observations, the students learned which physical properties of the Slinky allow for the bottom to remain stationary when the top was released. In particular, the students realized that the models must be very "stretchy" in order for the levitation phenomenon to be pronounced.[21]

B. Numerical exploration

Having convinced themselves that the discrete model can reproduce a phenomenon akin to that observed in the Slinky drop, the students next looked for deeper understanding of the Slinky drop in the mathematics of the model. Students were guided toward a numerical approach to solving the equations of the model that divided the evolution of the system into small time steps (e.g., explicit Euler method[22]). This was motivated by considering examples like the discrete frames in the slow-motion movies we used to view the Slinky. Discretizing time expanded on our previous decision to discretize mass in the Slinky.

The students then "simulated" the Slinky drop as a class. Four groups, each representing a mass of the discrete model, used their physical location in the classroom to act out the falling Slinky (Fig. 2). During every time step, the groups each calculated their change in position and velocity via the discretized Newton's law and then updated their locations in the classroom. They then shared their updated positions with the groups representing masses that they were connected to by springs. They also plotted their position on the main chalkboard, allowing the whole class to track the progress of the simulation. The communication of positions between groups was intended to foreshadow the inherent time delay in the passing of information, explored later in the week and described in the next section.

This numerical approach provides opportunities for students to iterate on their model as they confront unforeseen situations during the simulation. For example, the top mass

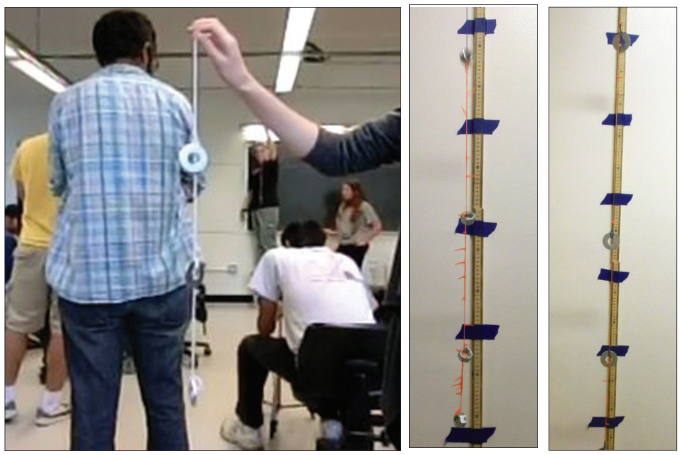

Fig. 1. Students from a recent Compass Project summer program testing model Slinkies built out of washers and rubber bands.

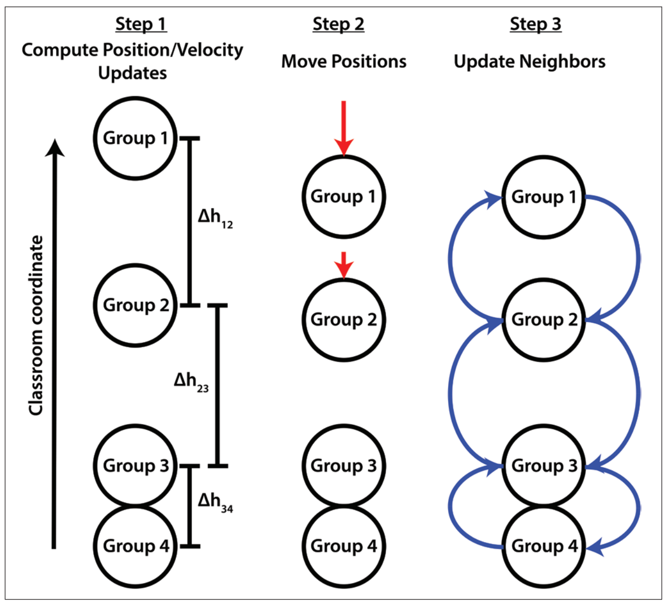

Fig. 2. Bird's-eye view diagram of classroom simulation. The movement of the discrete masses is modeled by groups of students moving across the classroom. Students alternate between computing position/velocity updates, moving across the classroom, and updating neighboring groups with new positions. The left column shows the layout of student groups in a classroom from above. Relative positions, masses, and spring constants are used to compute position and velocity updates using the Euler method. The middle column shows the groups physically moving across the classroom based on the updates. The right column shows students sharing updated positions with neighboring groups (those connected by springs).

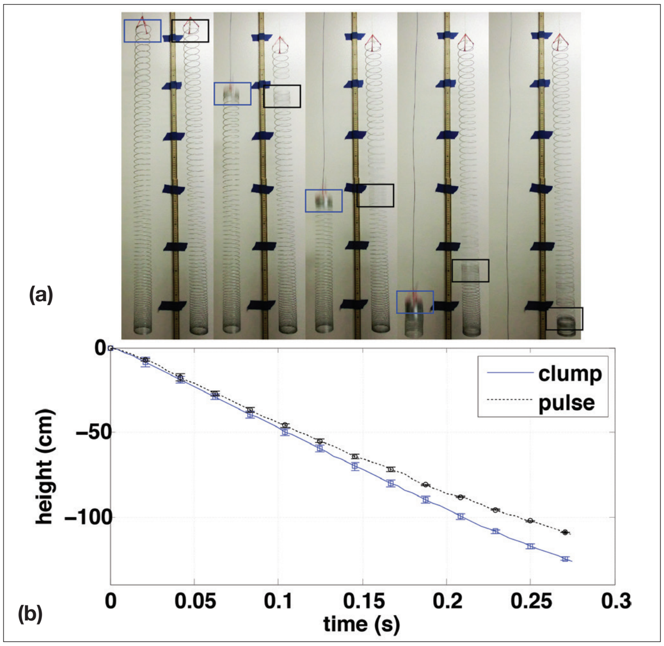

Fig. 3. (a) Five frames from a slow-motion image of a falling Slinky clump (left side of frames) next to a wave pulse sent down a Slinky (right side of frames). The clump is boxed in blue and the pulse in black. (b) A graph of position vs. time of the top of the falling Slinky clump and the wave pulse. Error bars correspond to uncertainty in annotation from multiple re-annotations as well as from multiple trials.

group will pass through the second mass group a few time steps into the simulation. The students may wish to allow this to happen, or they may wish to modify their procedure, perhaps by merging any two masses that have overlapped in the most recent time step in a way that conserves momentum.[23]

For the N-mass simulation used by the students (with $N = 4$), it will take $2N$ time steps for the bottom mass to have a non-zero position update. After some discussion, the students realized that this is not an explanation for the lack of motion in the real Slinky but is rather an artifact of the discretization of time and mass: the last mass will always remain stationary for $2N$ time steps regardless of the size of the time step. The fact that the time until the bottom mass moves depends on unphysical simulation parameters provides an opportunity to probe a limitation of this model, and motivates the exploration of an alternate approach for understanding the levitating Slinky phenomenon.

A wave model for information transfer across the Slinky

The limitations of the discrete model along with the role of communication between masses in the numerical simulation provided a nice transition to the next approach that models the Slinky as a medium for information transfer via waves. This model offers insight into the phenomenon of interest while managing to avoid much of the tedious intermediate calculations of the force-based approach. The information approach can be informally summed up in the following question: how does the bottom of the Slinky "know" that the top has been released?

Initially, using information in the physics context may seem ill defined and abstract for the students. We used a discussion of "information travel" in other contexts like earthquakes and tsunamis (displacement of Earth and water surface) or lightning and thunder (light and sound propagation) to motivate the idea of information as a fundamental entity that must be communicated from one place to another through a series of intermediate interactions.

In the context of vertically suspended Slinkies, students observed that information about disturbances at the top is carried by longitudinal waves to the bottom. The question then naturally presented itself of whether the "clump" of coils formed when the top of the Slinky is released would fall faster or slower than a wave pulse carrying this information. This motivated students to measure the dependence of the wave speed on parameters like the tension and density in a horizontally suspended Slinky (speed ~ $\sqrt{\text{tension/density}}$). Observing that the tension (which depends on the weight of coils below) and coil spacing (which is inversely proportional to density) decrease toward the bottom of a vertically suspended Slinky, the theory predicts that the wave will slow down as it travels closer to the bottom of the Slinky. It is thus plausible that the clump might reach the bottom of the Slinky before the wave can propagate all the way to the bottom.

Students took an experimental approach to compare the propagation times using two identical Slinkies that were suspended vertically next to each other. They permanently suspended one Slinky and initiated a wave pulse starting at the top. In this Slinky, the medium does not collapse, and only the pulse travels to the bottom. The other Slinky was suspended and then dropped from the top. In this Slinky, a clump of coils accumulates as the top of the Slinky falls to the bottom.[24] Using a slow-motion camera, the trajectory of the wave pulse and the clump can be qualitatively compared and quantitatively measured using a frame-by-frame analysis software.[25]

The results of the measurements are shown in Fig. 3. The data clearly show that the falling clump moves faster than a wave pulse would on the same Slinky, providing a very intuitive way of understanding the Slinky drop. Information travels at finite speed and so the news of the release of the Slinky will take time to propagate down the Slinky. Since the collapse speed of the medium exceeds the speed at which the wave pulse is capable of carrying the information, the bottom of the Slinky remains stationary until the top arrives.[1]

Conclusion

The curriculum presented is based around understanding why the Slinky drop seems to defy gravity. The fact that the Slinky drop lends itself so naturally to two very different models makes it an excellent tool for teaching model building. The first model represents the Slinky using a finite number of point masses and springs. The limitations of this discrete model motivate a second approach that models information transfer as waves through the Slinky and ultimately provides the key insight that the bottom of the Slinky does not initially "know" that the top has been released.

During a final discussion of the Slinky drop experiment, the students compared and contrasted the two models they had explored, noting that each model has its own strengths and makes different parts of the underlying physics more transparent. They considered which effects of the phenomenon were emphasized and what approximations had been made. The students reflected on the insights offered by each model along with the benefits of approaching a single problem from multiple angles. The Slinky drop experiment thus provided the ideal centerpiece for an accessible and hands-on curriculum intended to introduce students to model building in the physical sciences.

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Calvin Berggren, *Berkeley Compass Project, University of California, Berkeley, CA and Texas Lutheran University, Seguin, TX*

Punit Gandhi, *Berkeley Compass Project, University of California, Berkeley, CA and Mathematical Biosciences Institute, Ohio State University, Columbus, OH*
punit_gandhi@berkeley.edu

Jesse A. Livezey, *Berkeley Compass Project, University of California, Berkeley, CA and Lawrence Berkeley National Laboratory, Berkeley, CA*

Ryan Olf, *Berkeley Compass Project and Department of Physics, University of California, Berkeley, CA*